3ʳᵈ International NES Contest for High School Students

1 March — 11 April 2012

Problem Set

You are invited to solve 11 problems. Show the reasoning behind your answers for all problems. Make your arguments clear, logical and well-supported. All intermediate calculations must be shown. A correct answer without an explanation will receive minimum credit. While answering each question, remember to state any additional assumptions you make. Good luck!

Problem 1.   (5 points)

Mr. M., an entrepreneur, got an idea of how to make all people rich. He sent 10 letters to his friends suggesting for them to pay him 100 roubles each, and then in turn to mail 10 letters to their friends, suggesting for them to pay 100 roubles each to the mailer of the letter, and then to invite 10 of their friends to do the same, and so on... In the end, all participants will benefit from the deal: each one (except for Mr. M. himself) will pay 100 roubles, and receive 1000 roubles in return. Would this scheme allow all participants to benefit from it?

Problem 2.   (5 points)

An economist conducted a big research study. For each of the Russian regions he estimated the population size, employment rate, average income, average expenditures as well as some other demographic and socio-economic parameters. When asked what the average income in Russia was, he reported that it could be easily calculated by summing up all of the regional average incomes, and dividing the resulting sum by the number of regions. After giving it additional thought, the economist realized that this approach did not yield the correct estimate of the average income in Russia and suggested another method. Why does the originally proposed method produce an incorrect estimate? What method would you propose in order to answer the question based on the results of the conducted research?

Problem 3.   (5 points)

Sometimes people (firms, governments) engage in costly actions that do not yield them any immediate benefit. For example, the Spanish conquistador Aernan Kortes set his ships on fire after having arrived in Mexico; many countries have signed the Kyoto Treaty, promising to curb the level of harmful emissions produced into the atmosphere; people buy expensive gym memberships, knowing in advance that they will not be able to visit it often. What do all of these actions have in common? What is the explanation behind this type of behavior? Why do people act this way?

Problem 4. (7 points)

When a country's government proposes to raise the retirement age, people who are closer to retirement tend to protest the most. Middle-aged people and youth are more tolerant to such proposals.

a) Isn't that odd? The retirement age changes for all people, not only for those, who are the first to retire. How can you explain this reaction?

b) Experts proposed the following plan for increasing the retirement age by 5 years: the government should announce today that the retirement age will increase by 6 months every year during a 10-year period. What are the benefits and drawbacks of this approach?

Problem 5. (7 points)

The Ministry of Transportation has noticed that soon after energy tariffs were raised, the prices of railway transportation also rose, including both diesel and electric locomotives. In order to prevent the price growth, the Ministry suggested putting a price cap on energy tariffs for railroaders. Do you think this price cap will prevent the growth of prices for railroad transportation?

Problem 6. (7 points)

a) There are examples of Microsoft Office packages (composed of a text editor, spreadsheet, presentation editor, etc.) being priced differently, depending on the type of package. For example, "Office Home and Student" by Microsoft is composed of 4 basic programs and costs 7.5 times cheaper than the "Office Professional", which is composed of 7 programs (the 4 basic programs plus 3 additional ones). The programs included in the packages are not sold separately. Explain how the packaging of programs allows Microsoft to generate higher profits than the separate sales would have yielded.

b) Microsoft Corporation has an additional pricing policy feature: purchasing the Home and Student Office pack for three computers will cost you only 1.35 times more than purchasing the program for a single computer, not 3 times more as it would be expected. Explain how the per-unit discount on the purchase allows Microsoft to generate higher profits than the fixed price, independent from the volume of purchase.

Problem 7. (7 points)

USA has adopted various measures for discouraging smoking. One of the popular measures is prohibiting smoking in restaurants and bars. These smoking bans are mostly passed on the local level of cities, municipalities and counties. Research shows that banning smoking in restaurants and bars reduces prevalence of smoking as well as decreases the intensity of smoking amongst those who continue to smoke. This policy also had other unintended consequences: for instance, it led to the decline of alcohol consumption. However, the number of car accidents involving drunk drivers increased in the areas where restaurant and bar smoking bans were enacted. How would you explain the rise in the number of car accidents involving drunk drivers?

Problem 8. (7 points)

Eugene and Konstantin are participating in the program "Delicious Diets" and receive coupons which they can exchange for bottles of Coca-Cola and packs of chewing gum at the school's cafeteria. Eugene receives 2 coupons for which he can exchange 1 pack of chewing gum per coupon, and Konstantin receives 2 coupons for which he can exchange 1 bottle of Cola per coupon. However, both boys prefer to have one pack of chewing gum and one bottle of Cola, as they like the diversity. For both of them there is never too much chewing gum or Cola—the more they have of each, the better.

a) Can the boys engage in a mutually beneficial exchange? If so, what would the distribution of chewing gum and Cola look like?

b) Imagine that they can only make a sequential exchange: first Eugene decides whether he will give one pack of chewing gum to Konstantin, then Konstantin decides whether he will give Eugene a bottle of Cola. If Konstantin is in 7th grade, and Eugene is in 2nd grade and therefore can't claim back his coupon in case Konstantin tricks him, would Eugene still want to participate in the exchange? Would Konstantin?

c) Would the boys change their decisions if they agreed to make this exchange every day during one week? What if the agreement lasted for a whole year?

Problem 9. (14 points)

There are four cities on a plane, located at the vertices of a convex *ABCD* quadrilateral. The cities decided to host the Spring Olympics for the first time in history. However, neither of the cities has the required infrastructure to host the game, and each point on the plane (including the cities) can be chosen as a host of Spring Olympics. Each city would prefer the place of Spring Olympics to be as close to its location as possible.

a) We consider the location point of Olympics to be Socially Optimal if the sum of distances from each city to the location is minimal. Find the Socially Optimal Location and explain your choice.

b) Assume that the city may vote between two possible locations: *X* and *Y* (*X* and *Y* are not the same points). We say that point *X* wins over point *Y* if at least 3 out of 4 cities vote for *X*. Find the set of points *P* on the plane, such that there is no point distinct from *P* which wins over *P*. Are these points Socially Optimal?

Problem 10. (14 points)

The bank and currency regulation in the country of Fantia are very strict. There are 3 currencies in free circulation: Fantics (ф), Euros(€) and Dollars ($). All banks conduct the currency exchange at a single (but different for different pairs of currencies) exchange rate, set by the Central Bank. The following operations are allowed in Fantia:

1. **Currency Exchange.** Each citizen may exchange any amount of any currency he possesses to any other currency at the exchange rate, announced by Central Bank. Citizens are allowed to perform the exchange from multiple currencies that they posses or to multiple currencies during one operation of exchange. They are not allowed to perform more than 3 exchange operations during one day.

2. **Taking out a loan.** Each morning, before the exchange offices are opened, each citizen may take out a loan in any currency he wishes. He has to pay the loan back in the evening of the same day (after the exchange offices close). It is possible to take out a loan in any currency, but the loan amount may not exceed the amount of money possessed by the citizen in that particular currency. For example, if a citizen has 30 Euros and 70 Fantics, the maximum amount of Euros he can get is 30, and the maximum amount of Fantics he can get is 70.

3. **Opening a Deposit.** Each morning, before the Exchange Office is opened, citizens of Fantia can open a one-day deposit in any currency. The principal and the premium should be withdrawn from the deposit at the evening of the same day, after the Exchange Office is closed. Daily credit and deposit rates are presented in a table below:

currency	credit rate	deposit rate
€	1.5%	1%
$	2.25%	1.5%
ф	3%	2%

On Monday morning the Central Bank announces the following Exchange Rates for the next three days:

	Monday	Tuesday	Wednesday
ф for one €	43	43.4	43.1
ф for one $	33	32.9	33.6
$ for one €	1.28	1.36	1.3

For example, on Monday one can exchange 43 Fantics to 1 Euro and conversely 1 Euro to 43 Fantics. Imagine, you are a citizen of Fantia, and you have 100 Fantics in your pocket. What is the maximum amount of Fantics you can earn by Wednesday evening, if

a) It is prohibited to posses any foreign currency at the end of the day;

b) It is allowed to possess foreign currency, even at the end of the day.

Describe the algorithm of your actions and the amount of Fantics you will posses on Wednesday evening

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Problem 11. (22 points)

Three medical school students live in the 3-story dormitory, each occupying one floor. There is no elevator in the building. The students don't like dirt, so they decided to hire a janitor to clean the floor in front of each apartment.

A student's utility depends on the number of clean stair landings he passes on the way to his apartment. The more stair landings the student passes on his way home, the more tolerant he is to dirt.

The utility of a student occupying the i-th floor can be written as follows:

$$U_i = 20 - (4 - i) \cdot (i - x)^2 - 2y,$$

where x is the number of clean stair landings on his way home and y is the number of stair landings he pays the janitor to clean. In the case that the student receives the same utility from paying for cleaning the different number of stair landings, he prefers the option where he has to pay less.

The janitor agrees to clean one stair landing for 2 dollars. She always starts from the first floor, moving to the second floor, and then cleaning the 3rd floor, regardless of who is paying. Hence, the student cant make her clean his stair landing before she has cleaned all the lower ones. For example, for 4 dollars she will clean only the 1st and 2nd floors, regardless of who has paid her.

At the beginning of the month every student announces how many stair landing cleanings he is willing to sponsor. Each student may pay for any whole number of landings. Students make their commitments simultaneously. We call the set of students commitments *an equilibrium* if neither student is better off by changing his choice after the promises are announced.

a) How many floors will be cleaned in equilibrium? Who would pay for the cleaning?

b) The janitor has changed her habits, and now she starts cleaning from the top floor (she cleans the 3rd floor first, the 2nd floor — second, and the 1st floor is cleaned the last). How would the answers to the question change?

c) After cleaning the house for a while, the janitor noticed that the first floor is always the dirtiest, as three people pass through it. The 2nd floor is 1.5 times cleaner, and the 3rd floor is 3 times cleaner than the 1st floor. She decided that she would still charge 6 dollars for cleaning the whole house, but now she charge each floor proportionally to its dirtiness. The cleaning of the 1st floor would now cost 3 dollars, 2nd floor would cost 2 dollars, and the cleaning of the 3rd floor would only cost 1 dollar. Hence, the utility of the student living on the i-th floor can now be written as follows:

$$U_i = 20 - (4 - i) \cdot (i - x)^2 - z,$$

where z is the share of cleaning that was sponsored by the student (out of 6 dollars), all other notations stay the same. How would the answers to the questions a) and b) change in this case?

d) Assume now that the amount to be paid to the janitor is set not by the students, but by the managing company. The company wants to maximize the joint utility of all students. What outcomes would the conditions of question a), b), and c) yield? How would the joint utility of students change in cases a), b), and c), if the managing company is present.